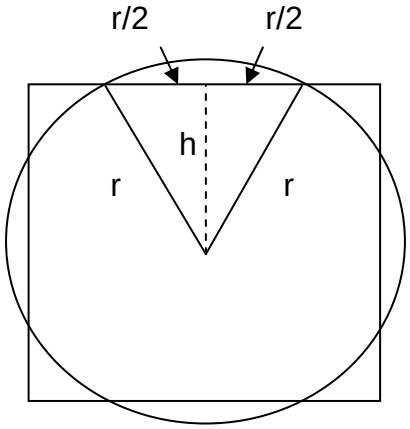
University of North Georgia

Sophomore Level Mathematics Tournament

April 6, 2013

Solutions for the Afternoon Team Competition

Round 1



To find the area of the square:

Let $h = \frac{1}{2}$ the length of the square.

Then $h^2 + \left(\frac{r}{2}\right)^2 = r^2$, and $h = \frac{\sqrt{3}}{2} r$.

So the length of the square $2h = \sqrt{3} r$.

Thus, the area of the square $\left(\sqrt{3} r\right)^2 = 3r^2$.

The area of a circle $= r^2$.

So, $\frac{\text{Area of square}}{\text{Area of circle}} = \frac{3r^2}{r^2} = \frac{3}{\pi}$ or 3 : π.

Round 2

The TI calculator gives 1.524157875E16 as the answer. One way to work this out by hand is:

$(123,450,000 - 6,789)(123,450,000 - 6,789)$

$15,239,902,500,000,000 - 838,102,050,000 - 838,102,050,000 + 46,090,521$

$15,241,578,750,190,521$

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Round 3

Let D be the distance (in miles) between New Orleans and St. Louis, x be the speed of the ship with respect to the water (in miles per week), and y be the speed of the current. The equations are:

$$\begin{cases} D = 2(x+y) \\ D = 3(x-y) \end{cases}$$

Distribute to get rid of parentheses and divide both sides by y, giving:

$$\begin{cases} \frac{D}{y} = \frac{2x}{y} + 2 \\ \frac{D}{y} = \frac{3x}{y} - 3 \end{cases}$$

Multiply the first equation by 3 and the second by -2. Then add the two equations, giving:

$$\frac{D}{y} = 12$$

Since D is the distance (in miles), and y the speed of the raft (in miles per week), then $\frac{D}{y} = 12$ is the time (in weeks).
It will take 12 weeks for the raft to reach New Orleans.

Round 4

$$(x+a)^2 = (x+a)(x+a) = x^2 + 2ax + a^2$$

$$(x+a)^2(x+a) = (x^2 + 2ax + a^2)(x+a) = x^3 + 3ax^2 + 3a^2x + a^3$$

$$(x+a)^3(x+a) = (x^3 + 3ax^2 + 3a^2x + a^3)(x+a) = x^4 + 4ax^3 + 6a^2x^2 + 4a^3x + a^4$$

$$(x+a)^4(x+a) = x^5 + 5ax^4 + 10a^2x^3 + 10a^3x^2 + 5a^4x + a^5$$

Thus $5a = 10a^2$.

Since $a > 0$, $a = \frac{1}{2}$.

Round 5

For short, let's use the first letters of the names of animals, instead the whole names.

The solution will contain two steps:

1. First it will show that each arrangement of cages has to contain at least four cages.
2. Then it will show that it is possible to arrange animals in four cages without them harming each other.

Step 1: T and L cannot be together (the first row of the table), so any arrangement will have to place them in two separate cages.

Now consider B. It cannot be with L (the second row of the table) and cannot be with T (the fourth row of the table). So there are at least three different cages required to house T, L, and B.

Now consider S. According to the last row of the table, S cannot be with T, L, and B. Thus there are at least four different cages required in any arrangement.

Step 2: Here is a solution that places animals in four cages in such a way that they will not harm each other:

T, C

c a

Round 8

Since the area of the circle is ^{2}

Round 10

We know that $\text{speed} = \frac{\text{distance}}{\text{time}}$, so $t = \text{time} = \frac{\text{distance}}{\text{speed}}$.

Let the cyclist go U miles uphill at 6 mph, then $t_{up} = \frac{U}{6}$ hrs.

Let the cyclist go D miles downhill at 12 mph, then $t_{down} = \frac{D}{12}$ hrs.

Let the cyclist go L miles on level at 8 mph, then $t_{level} = \frac{L}{8}$ hrs.

The total time from town M to town $N = 4$ hrs, so

$$t_{up} + t_{down} + t_{level} = 4$$

$$\frac{U}{6} + \frac{D}{12} + \frac{L}{8} = 4 \qquad (1)$$

On the return trip, up becomes down and down becomes up, so

$$\frac{D}{6} + \frac{U}{12} + \frac{L}{8} = 4.5 \qquad (2)$$

Multiply (1) by 24 and multiply (2) by 24 to get:

$$-4U + 2D + 3L = 96$$
$$\oplus\ 4D + 2U + 3L = 108$$

Add the equations to get:

$$6U + 6D + 6L = 204$$
$$U + D + L = \frac{204}{6} = 34$$

So the distance from town M to town $N = 34$ miles