Solutions for the Afternoon Team Competition

Round 1

There are 3 different choices for Andrea. Since Bob and Andrea can't have vacation during the same month, there are 2 choices for Bob. Dave must also have a different month of vacation than Andrea. Carrie can't have the same month of vacation as Bob or Dave. If Bob and Dave both have vacation the same month, then there are two choices for Carrie's month of vacation. This gives: $3 \cdot 2 \cdot 2 \cdot 1 = 12$.
If Bob and Dave have different months of vacation, then there is one choice for Carrie's month
` Round 2

Using the remainder theorem; $2^5, 9, 23$.

Round 3

The possibilities are:
$2 \cdot 3 \cdot 5 \cdot 7 \cdot 11 \cdot 23 = 10$
$2 \cdot 3 \cdot 5 \cdot 7 \cdot 13 \cdot 27 = 30$
$2 \cdot 3 \cdot 5 \cdot 7 \cdot 17 \cdot 35 = 70$
$2 \cdot 3 \cdot 5 \cdot 7 \cdot 19 \cdot 39 = 90$
$2 \cdot 3 \cdot 5 \cdot 7 \cdot 23 \cdot 48 = 30$
$2 \cdot 3 \cdot 5 \cdot 11 \cdot 13 \cdot 429 = ?$

So, there are 6 possible numbers.

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Eighteenth Annual Mathematics Tournament

April 14, 2012

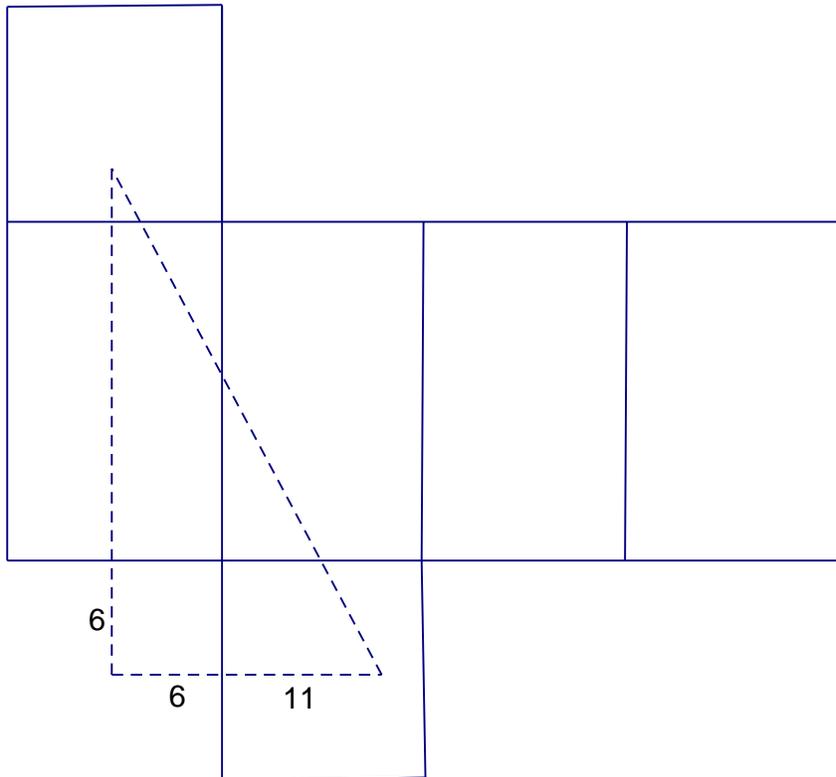## Round 4

The angle opposite the $80°$ angle is also $80°$. Let $\angle 1 = \angle 2 = x$.
Then $80 + 2x = 180$ and $x = 50$ so, $\angle 1 = 50°$.
So, $\angle DBA = 180° - \angle DAB - \angle 1 = 180° - 103° - 50° = 27°$.
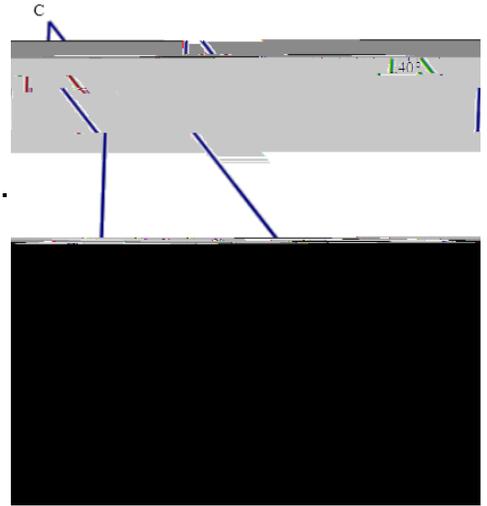
Round 5

Consider the diagram below of a net of the room with S indicating the initial position of the spider and F indicating the initial location of the fly.

This adds up to 18. There are also the following two squares with vertices: (0, 1) (1, 2) (2, 1) (1, 0) and (0, 2) (1, 3) (2, 2) (1, 1)
So there are a total of 20 rectangles.

Round 9

Let S be the swimmer's speed in still water and let C be the speed of the current.

Suppose the swimmer swims against the current at a rate of S – C for 10 minutes. Then his distance traveled during that time would be (S – C)10.

Then suppose the swimmer swims downstream at a rate of S + C for x minutes. Then his distance traveled during that time would be (S + C)

Since the second distance is 1000 yards longer, (S + C) = (S – C)10 = 1000

During this time, the hat has floated 1000 yards at a rate of C. So C(10 + x) = 1000

So there are two equations:

Sx + Cx – 10S + 10C = 1000 or if you rearrange the terms Sx – 10S + 10C + Cx = 1000 and

10C + Cx = 1000

Substitute 1000 into the first equation for 10C + Cx

Sx – 10S + 1000 = 1000

Sx = 10S
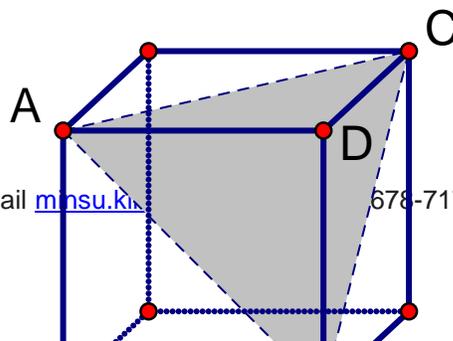
x = 10

Substitute into the equation above

10C + C(10) = 1000

20C = 1000

C = 50 yards per minute The speed of the current is 50 yards per minute.

Round 10

The pyramid can be viewed in such a way that the base is a right triangle that is half of one of the cube's sides.

V = (1/3)(area of base)(height)

$$972 = \frac{1}{3}\left(\frac{1}{2}s^2\right)s$$

$$972 = \frac{1}{6}s^3$$

$$5832 = s^3$$

$s = 18$ meters